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On the entropy of four-dimensional near-extremal $N = 2$ black holes with R^2-terms

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Abstract: We consider the entropy of four-dimensional near-extremal $N = 2$ black holes. The Bekenstein-Hawking entropy formula has the structure of the extremal black holes entropy with a shift of the charges depending on the non-extremality parameter and the moduli at infinity. We construct a class of near-extremal horizon solutions with R^2-terms, and show that the generalized Wald entropy formula exhibits the same property.

Keywords: Black Holes in String Theory, Black Holes, Supergravity Models.

Contents

1. Introduction

Explaining the microscopic origin of black holes entropy is one of the important tasks of any theory of quantum gravity. Much progress towards achieving this goal in the framework of string theory has been obtained for supersymmetric charged black holes in various dimensions. One such class of extremal black holes characterized by electric and magnetic charges (q_I, p^I) exists in compactification of type II string theory on Calabi-Yau 3-folds (CY_3) (for a review see [1, 2]). These black holes are obtained by wrapping D-branes around cycles in CY_3. Their near horizon geometry is $AdS_2 \times S^2 \times CY_3$, where the moduli of the Calabi-Yau 3-fold are fixed on the horizon by the attractor equation in terms of the charges. Recently much work has been done on extremal non-supersymmetric black holes [3].

The four-dimensional low energy effective action of type II strings compactified on CY_3 is given by $N = 2$ Poincaré supergravity coupled to $N = 2$ abelian vector multiplets. The macroscopic extremal black holes are asymptotically flat charged supersymmetric solutions of the field equations. At leading order in the curvature, the entropy of the black holes is given by the Bekenstein-Hawking area law $S = \frac{A}{4}$, where A is the area of the event horizon and is determined in terms of the charges by the attractor mechanism. With subleading R^2-terms included, the entropy of these macroscopic black holes has been computed using the generalized entropy formula of Wald [4].

With one electric charge q_0 and p^A magnetic charges[1] one gets the entropy [5]

$$S = 2\pi \sqrt{q_0 \left(D_{ABC} p^A p^B p^C + 256 D_A p^A \right)}, \tag{1.1}$$

[1] We consider type II compactification with $A = 1, \ldots, b_2$ and b_2 is the second Betti number of CY_3.

where D_{ABC} and D_A are respectively proportional to the triple intersection numbers and the second Chern class numbers of the CY_3. The first term in (1.1) is the Bekenstein-Hawking area law, while the second term is the R^2 generalized entropy formula correction.

The aim of this paper is to study near-extremal $N = 2$ black hole solutions and their entropy with R^2-terms included. These are non-supersymmetric solutions, the horizon is no longer $AdS_2 \times S^2$ and the attractor mechanism no longer works. The moduli of the CY_3 are not fixed at the horizon in terms of the charges and the entropy depends on the asymptotic values of the moduli. However, when considering the Bekenstein-Hawking entropy one notices that it has the same structure as that of the extremal ones with charges being shifted [6, 7]

$$q_0 \to q_0 + \frac{1}{2}\mu h_0, \qquad p^A \to p^A + \frac{1}{2}\mu h^A , \tag{1.2}$$

where μ is the non-extremality parameter and (h_0, h^A) correspond to the asymptotic values of the moduli. A natural question to ask is whether this property of the near-extremal Bekenstein-Hawking entropy holds with the R^2 generalized entropy formula (1.1). Indeed, we will provide evidence that this is the case for a class of near-extremal charged black holes with $D_{ABC}p^A p^B p^C = 0$, i.e. having an extremal limit with a vanishing classical horizon area.

A relation between the indexed entropy of the BPS $N = 2$ black holes and the topological string partition function, evaluated at the attractor point has been proposed in [8]

$$Z_{\mathrm{BH}} = |Z_{\mathrm{top}}|^2 \ . \tag{1.3}$$

We suggest that one may still use the relation (1.3) for this class of near-extremal $N = 2$ black holes, with the shift in the charges (1.2). If correct, one gets all the perturbative F-terms corrections to the near-extremal $N = 2$ black holes entropy using the topological string partition function. We note that this is unlikely to be correct for general near-extremal $N = 2$ black holes. Indeed, we find near-extremal R^2 horizon solutions with $D_{ABC}p^A p^B p^C \neq 0$, which do not exhibit this entropy structure.

The paper is organized as follows: In section 2 we will give a brief review of four-dimensional $N = 2$ supergravity with R^2-terms. In section 3 we derive a generalized Wald entropy formula for the near-extremal $N = 2$ black holes with R^2-terms. In section 4 we present the horizon geometry of near-extremal $N = 2$ black hole and compute the entropy. We will first review previous results without R^2-terms, and then present the new results with R^2-terms.

In the paper we will use $a = 0, 1, 2, 3$ to denote the tangent space indices corresponding to the indices μ of the space-time coordinates (t, r, ϕ, θ).

2. R^2-terms in $N = 2$ supergravity — A brief review

We will consider $N = 2$ Poincaré supergravity coupled to N_V abelian $N = 2$ vector multiplets. $N = 2$ Poincaré supergravity can be formulated as a gauge fixed version of $N = 2$ conformal supergravity coupled to an $N = 2$ abelian vector multiplet (see [1] for a comprehensive review).

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The on-shell field content of the vector multiplet is a complex scalar, a doublet of Weyl fermions, and a vector gauge field. We will consider $N_V + 1$ vector multiplets, and will denote by X^I, $I = 0 \dots N_V$, the scalars (moduli) in the vector multiplets. The couplings of the vector multiplets are encoded in a prepotential $F(X^I)$, which is a homogenous of second degree holomorphic function.

The $N = 2$ conformal supergravity multiplet (Weyl multiplet) is denoted by W^{abij}, where i, j are SU(2) indices. It consists of gauge fields for the local symmetries: translations (P), Lorentz transformations (M), dilatations (D), special conformal transformations (K), U(1) transformations (A), SU(2) transformations (V), and supertransformations (Q, S). In the theory without R^2-terms, the Weyl multiplet appears in the Lagrangian through the superconformal covariantizations. In order to get the R^2-terms, one adds explicit couplings to the Weyl multiplet. This appears in the form of a chiral multiplet, which is equal to the square of the Weyl multiplet W^2. The lowest component of the chiral multiplet is a complex scalar denoted $\widehat{A}$. The prepotential $F(X^I, \widehat{A})$ describes the coupling of the vector multiplets and the chiral multiplet.

We consider a prepotential of the form:

$$F = \frac{D_{ABC} X^A X^B X^C}{X^0} + \frac{D_A X^A}{X^0} \widehat{A}\,, \tag{2.1}$$

where D_{ABC}, D_A are constants and $A, B, C = 1 \dots N_V$. This prepotential arises, for instance, from a compactification of Type IIA string theory on a Calabi-Yau three-fold. The coefficients in the prepotential are topological data of the Calabi-Yau three-fold: $-6D_{ABC}$ are the triple intersection numbers (symmetric in all indices), and $-1536 D_A$ are the second Chern class numbers. The first term in the prepotential arises at tree-level in α' and in g_s. The second term arises at tree-level in α' and is at one-loop in g_s. It describes R^2 couplings in the Lagrangian. In the large Calabi-Yau volume approximation $\mathrm{Im}(X^A/X^0) \gg 1$, all other corrections are suppressed and the prepotential consists of only these two terms. We will assume this approximation to be valid near the horizon by an appropriate hierarchy of charges. The equations of motion should later be truncated to the same order of approximation. One introduces the notation

$$F_I \equiv \frac{\partial}{\partial X^I} F(X^I, \widehat{A}), \qquad F_{\widehat{A}} \equiv \frac{\partial}{\partial \widehat{A}} F(X^I, \widehat{A})\,, \tag{2.2}$$

and similarly for higher order and mixed derivatives.

The bosonic part of the $N = 2$ conformal supergravity Lagrangian is

$$\begin{aligned} 8\pi e^{-1}\mathcal{L} = & -\frac{i}{2}(\bar{X}^I F_I - X^I \bar{F}_I) R \\ & + \left(i\mathcal{D}^a \bar{X}^I \mathcal{D}_a F_I + \frac{i}{4} F_{IJ} (F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T^-_{ab}) \left(F^{ab-J} - \frac{1}{4}\bar{X}^J T^{ab-} \right) \right. \\ & + \frac{i}{8}\bar{F}_I \left(F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T^-_{ab} \right) T^{ab-} + \frac{i}{32}\bar{F} T^-_{ab} T^{ab-} - \frac{i}{8} F_{IJ} Y^I_{ij} Y^{ijJ} \\ & - \frac{i}{8} F_{\widehat{A}\widehat{A}} (\varepsilon^{ik}\varepsilon^{jl} \widehat{B}_{ij} \widehat{B}_{kl} - 2\widehat{F}^-_{ab} \widehat{F}^{ab-}) + \frac{i}{2}\widehat{F}^{ab-} F_{\widehat{A}I} \left(F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T^-_{ab} \right) \end{aligned} \tag{2.3}$$

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$$
\begin{aligned}
&-\frac{i}{4}\widehat{B}_{ij}F_{\widehat{A}I}Y^{ijI}+\frac{i}{2}F_{\widehat{A}}\widehat{C}+\text{h.c.}\Big) \\
&+i(\bar{X}^I F_I - X^I \bar{F}_I)\Big(\mathcal{D}^a V_a - \frac{1}{2}V^a V_a - \frac{1}{4}M_{ij}\bar{M}^{ij} + D^a \Phi^i{}_\alpha D_a \Phi^\alpha{}_i\Big)\,.
\end{aligned}
$$

$e \equiv \sqrt{|\det(g_{\mu\nu})|}$ where $g_{\mu\nu}$ is the curved metric, R is the Ricci scalar, D_a is the covariant derivative with respect to all superconformal transformations, $\mathcal{D}_a$ is the covariant derivative with respect to P, M, D, A, V-transformations, F_{ab}^{-I} is the anti-selfdual part of the vector field strength, T_{ab}^- is an anti-selfdual antisymmetric auxiliary field of the Weyl multiplet, $i, j, \ldots = 1, 2$ are SU(2) indices, Y_{ij}^I are auxiliary scalars of the vector multiplet, and $V^a, M_{ij}, \Phi^i{}_\alpha$[2] are components of a compensating nonlinear multiplet with $\alpha = 1, 2$. The hatted fields are components of the chiral multiplet W^2, with their bosonic parts given by

$$
\begin{array}{lrcl}
\theta^0 & \widehat{A} &=& T_{ab}^- T^{ab-} \\
\theta^2 & \widehat{B}_{ij} &=& -16\varepsilon_{k(i}R(V)^k{}_{j)ab}T^{ab-} \\
 & \widehat{F}^{ab-} &=& -16\mathcal{R}(M)_{cd}{}^{ab}T^{cd-} \\
\theta^4 & \widehat{C} &=& 64\mathcal{R}(M)_{cd}{}^{ab-}\mathcal{R}(M)^{cd-}{}_{ab} + 32R(V)_{ab}{}^{k-}{}_l\, R(V)^{abl-}{}_k - 16T^{ab-}D_a D^c T_{cb}^+\,.
\end{array}
\tag{2.4}
$$

$T_{ab}^+ = \bar{T}_{ab}^-$ is the selfdual counterpart of the auxiliary field, $R(V)_{ab}{}^k{}_l$ is the field strength of the SU(2) transformations, $\mathcal{R}(M)_{ab}{}^{cd}$ is the modified Riemann curvature and $\mathcal{R}(M)_{ab}{}^{cd-}$ is the anti-selfdual projection in both pairs of indices. The bosonic part of $\mathcal{R}(M)_{ab}{}^{cd}$ is given by[3]

$$
\mathcal{R}(M)_{ab}{}^{cd} = R_{ab}{}^{cd} - 4f_{[a}{}^{[c}\delta_{b]}^{d]} + \frac{1}{32}(T_{ab}^- T^{cd+} + T_{ab}^+ T^{cd-})\,, \tag{2.5}
$$

where $R_{ab}{}^{cd}$ is the Riemann tensor, and $f_a{}^c$ is the connection of the special conformal transformations, determined by the conformal supergravity conventional constraints, with the bosonic part:

$$
f_a{}^c = \frac{1}{2}R_a{}^c - \frac{1}{4}\left(D + \frac{1}{3}R\right)\delta_a^c - \frac{i}{2}{}^\star R(A)_a{}^c + \frac{1}{32}T_{ab}^- T^{cb+}\,, \tag{2.6}
$$

where $R_a{}^c$ is the Ricci tensor, D is an auxiliary real scalar field of the Weyl multiplet, and ${}^\star R(A)^a{}_b$ is the Hodge dual of the field strength of the U(1) transformations. Note that the T^2-terms in $\mathcal{R}(M)_{ab}{}^{cd}$ cancel exactly the T^2 contribution from $f_a{}^c$.

The auxiliary field D is constrained by a constraint on the nonlinear multiplet:

$$
\mathcal{D}^a V_a - D - \frac{1}{3}R - \frac{1}{2}V^a V_a - \frac{1}{4}M_{ij}\bar{M}^{ij} + D^a\Phi^i{}_\alpha D_a \Phi^\alpha{}_i = 0\,, \tag{2.7}
$$

where we have assumed a bosonic solution. Note that taking the constraints into account we have

$$
\frac{\partial f_e{}^f}{\partial R_{ab}{}^{cd}} \sim \frac{1}{2}\delta_e^a \delta_c^f \delta_d^b\,, \tag{2.8}
$$

where the r.h.s. must be constrained to have the same symmetries as the l.h.s. .

[2] $\Phi^\alpha{}_i$ is the hermitian conjugate of $\Phi^i{}_\alpha$.

[3] We have assumed the K-gauge fixing which will be defined later (2.9).

In order to obtain Poincaré supergravity one gauge fixes the bosonic fields (in addition there is a gauge fixing of fermionic fields):

$$\begin{aligned} K\text{−gauge}:&\quad b_a = 0 \\ D\text{−gauge}:&\quad i(\bar{X}^I F_I - X^I \bar{F}_I) = 1 \\ A\text{−gauge}:&\quad X^0 = \bar{X}^0 > 0 \\ V\text{−gauge}:&\quad \Phi^i{}_\alpha = \delta^i_\alpha\,, \end{aligned} \tag{2.9}$$

where b_a is the connection of the dilatations.

In addition to F-terms corrections to the $N = 2$ effective Lagrangian, one generally expects D-term corrections which may be relevant already in the R^2-level. For supersymmetric black holes, it is conjectured that such terms do not contribute to the entropy [8].

3. Entropy formula with R^2-terms

With the addition of R^2-terms to the Lagrangian, the Bekenstein-Hawking entropy formula is no longer valid. A generalization of the area law has been derived by Wald [4]. The Bekenstein-Hawking area is recovered when taking the Einstein-Hilbert Lagrangian. We work with the $N = 2$ supergravity Lagrangian, which does not depend on derivatives of the Riemann tensor, and we further assume the black holes to be static and spherically symmetric. The generalized entropy formula in this case is

$$S = 2\pi A \varepsilon_{ab}\varepsilon^{cd} \frac{\partial(e^{-1}\mathcal{L})}{\partial R_{ab}{}^{cd}}\,, \tag{3.1}$$

where A is the (modified) area of the horizon, $\varepsilon_{01} = -\varepsilon_{10} = 1$, $\mathcal{L}$ is the Lagrangian density, and the expression is evaluated on the event horizon. In the derivative, we treat the Riemann tensor and metric as being independent and take into account the supergravity constraints on the fields. Our derivation is similar to that of [5, 1].

We get

$$\begin{aligned} \frac{\partial(e^{-1}\mathcal{L})}{\partial R_{ab}{}^{cd}} &= -\frac{1}{16\pi}\delta^a_c\delta^b_d + \\ &\quad -\frac{1}{8\pi}\mathrm{Im}\left(F_{\widehat{A}I}\left(F^{-I}_{ef} - \frac{1}{4}\bar{X}^I T^-_{ef}\right)\frac{\partial \widehat{F}^{ef-}}{\partial R_{ab}{}^{cd}} + F_{\widehat{A}\widehat{A}}\widehat{F}^-_{ef}\frac{\partial \widehat{F}^{ef-}}{\partial R_{ab}{}^{cd}} + F_{\widehat{A}}\frac{\partial \widehat{C}}{\partial R_{ab}{}^{cd}}\right)\,. \end{aligned}$$

This expression can be simplified to:[4]

$$\begin{aligned} \frac{\partial(e^{-1}\mathcal{L})}{\partial R_{ab}{}^{cd}} &= -\frac{1}{16\pi}\delta^a_c\delta^b_d + \frac{1}{\pi}\mathrm{Im}\Bigg[\Bigg(2F_{\widehat{A}I}\left(F^{-I}_{pq} - \frac{1}{4}\bar{X}^I T^-_{pq}\right)T^{mn-} - 32F_{\widehat{A}\widehat{A}}\mathcal{R}(M)^{xy}{}_{pq}T^-_{xy}T^{mn-} \\ &\quad -16F_{\widehat{A}}\mathcal{R}(M)^{mn-}{}_{pq}\Bigg)\frac{\partial \mathcal{R}(M)_{mn}{}^{pq}}{\partial R_{ab}{}^{cd}} - F_{\widehat{A}}T^{an-}T^+_{cn}\delta^b_d\Bigg]\,, \end{aligned} \tag{3.2}$$

[4]Using the identity for anti-selfdual tensors: $R^{mn-}{}_{pq}R_{mn}{}^{pq-} = R^{mn-}{}_{pq}R_{mn}{}^{pq}$.

where $m, n, \ldots = 0 \ldots 3$. The last term is the same as in the supersymmetric case.[5]

We have the relation:

$$\mathcal{R}(M)^{mn}{}_{pq} = C^{mn}{}_{pq} + D\delta^{[m}_{[p}\delta^{n]}_{q]} - 2\delta^{[m}_{[p}{}^{\star}R(A)^{n]}{}_{q]}\,, \tag{3.3}$$

where $C^{ab}{}_{cd}$ is the Weyl tensor. In addition, from the definition of $\mathcal{R}(M)_{ab}{}^{cd}$ we get:

$$\frac{\partial \mathcal{R}(M)_{mn}{}^{pq}}{\partial R_{ab}{}^{cd}} \sim \delta^a_m \delta^b_n \delta^p_c \delta^q_d - 2\delta^{[p}_{[m}\delta^a_{n]}\delta^{q]}_c \delta^b_d\,, \tag{3.4}$$

where the r.h.s. must be constrained to have the same symmetries as the l.h.s., and we used $D = -\frac{1}{3}R + \ldots$ due to the nonlinear multiplet constraint (2.7).

Substituting all expressions, we obtain the generalized entropy formula for the non-extremal R^2 case:

$$S = \frac{1}{4}A - 4A \cdot \mathrm{Im}\Big(F_{\widehat{A}}(|T^-_{01}|^2 + 16C_{0101} + 16D)\Big)\,, \tag{3.5}$$

where we have used spherical symmetry, everything is evaluated on the event horizon, and $\widehat{A} = -4(T^-_{01})^2$. This formula differs from the extremal R^2 case by the C_{0101} and D terms, where also

$$\widehat{A} = -256\pi A^{-1}\ . \tag{3.6}$$

Note that as in the extremal R^2 case, the entropy does not depend on the higher order derivatives $F_{\widehat{A}I}, F_{\widehat{A}\widehat{A}}$.

4. Near-extremal $N = 2$ black holes

4.1 Near-extremal $N = 2$ black holes without R^2-terms

We will start by discussing non-extremal black holes in $N = 2$ supergravity without R^2-terms [6, 7]. The metric is given by

$$ds^2 = -e^{-2U(r)}f(r)dt^2 + e^{2U(r)}(f(r)^{-1}dr^2 + r^2 d\Omega^2)\,, \tag{4.1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and

$$f(r) = 1 - \frac{\mu}{r}\,, \tag{4.2}$$

and μ is a non-extremality parameter.

The background is non-supersymmetric, with $\frac{1}{2}\mu$ being the difference between the ADM mass and the BPS mass. The event horizon is located at $r = \mu$ and the inner horizon at $r = 0$. Unlike the extremal black holes, the event horizon geometry is not $AdS_2 \times S^2$.

[5]Using that $D_a D^c T^+_{cb} = \mathcal{D}_a \mathcal{D}^c T^+_{cb} - f_a{}^c T^+_{cb}$, for a bosonic solution. The covariant derivative string may be expanded as $\mathcal{D}_a \mathcal{D}^c = \frac{1}{2}\{\mathcal{D}_a, \mathcal{D}^c\} + \frac{1}{2}[\mathcal{D}_a, \mathcal{D}^c]$. Only the anticommutator part is dependent on the Riemann tensor, however its contribution vanishes due to the identity for (anti-)selfdual tensors: $T^{ab-}T^{c+}_{\ \ b} = T^{cb-}T^{a+}_{\ \ b}$.

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Consider the prepotential:

$$F = \frac{D_{ABC} X^A X^B X^C}{X^0}\,, \tag{4.3}$$

and the ansatz

$$e^{2U(r)} = e^{-K}\,, \tag{4.4}$$

where the Kähler potential K is

$$e^{-K} = i\Big(\bar{X}^I(\bar{z}) F_I(z) - X^I(z)\bar{F}_I(\bar{z})\Big)\,. \tag{4.5}$$

$F_I(z) = F_I(X(z))$ and $X^I(z)$ are related to the X^I, by

$$X^I = e^{\frac{1}{2}K} X^I(z)\,. \tag{4.6}$$

Consider black holes with one electric charge q_0 and $p^A, A = 1,2,3$ magnetic charges. One introduces the boost parameters γ^A, γ_0, related to the charges by

$$\begin{aligned} p^A &= h^A \mu \sinh\gamma^A \cosh\gamma^A \qquad \text{(no summation)} \\ q_0 &= h_0 \mu \sinh\gamma_0 \cosh\gamma_0\,, \end{aligned} \tag{4.7}$$

where h^A, h_0 are constants[6] that determine the moduli at infinity. Note that for fixed charges and non-extremality parameter, a choice of (γ^A, γ_0) is equivalent to a choice of (h^A, h_0). The extremal case is recovered in the limit $\mu \to 0; \gamma^A, \gamma_0 \to \infty$, with the charges held fixed.

Introduce the modified charges

$$\begin{aligned} \tilde{p}^A &\equiv h^A \mu \sinh^2\gamma^A = \alpha^A p^A \qquad \text{(no summation)} \\ \tilde{q}_0 &\equiv h_0 \mu \sinh^2\gamma_0 = \alpha_0 q_0\,, \end{aligned} \tag{4.8}$$

where $\alpha^A \equiv \tanh\gamma^A, \alpha_0 \equiv \tanh\gamma_0$. In the extremal case $\alpha^A, \alpha_0 \to 1$.

In the extremal supersymmetric case, the vanishing of the gaugino variations under $N = 1$ supertransformations, implies generalized stabilization equations, also called the supersymmetric attractor mechanism [9]. These equations determine the values of the moduli on the horizon in terms of the electric and magnetic charges. In the non-extremal case the gaugino variations do not vanish. Consider an ansatz similar to the supersymmetric stabilization equations of the form

$$\begin{aligned} i(X^I(z) - \bar{X}^I(\bar{z})) &= \tilde{H}^I \\ i(F_I(z) - \bar{F}_I(\bar{z})) &= \tilde{H}_I\,, \end{aligned} \tag{4.9}$$

where $\tilde{H}^I, \tilde{H}_I$ are harmonic functions

$$\begin{aligned} \tilde{H}^I &= h^I + \frac{\tilde{p}^I}{r} \\ \tilde{H}_I &= h_I + \frac{\tilde{q}_I}{r}\,. \end{aligned} \tag{4.10}$$

[6]These parameters are constrained by the asymptotic flatness condition: $e^{2U(\infty)} = |h^I F_I(\infty) - h_I X^I(\infty)|^2 = 1$.

These equations do not exhibit an attractor behavior, since the moduli on the event horizon at $r = \mu$ depend on the moduli at infinity.

The auxiliary field T^-_{ab} takes the form

$$T^-_{01} = iT^-_{23} = \left(\frac{k_0}{\alpha_0(r+k_0)} + \frac{k^1}{\alpha^1(r+k^1)} + \frac{k^2}{\alpha^2(r+k^2)} + \frac{k^3}{\alpha^3(r+k^3)}\right)\frac{1}{r}e^{-U(r)}\,, \qquad (4.11)$$

where

$$\begin{aligned} k^A &\equiv \mu \sinh^2\gamma^A = \frac{\mu(\alpha^A)^2}{1-(\alpha^A)^2} \qquad \text{(no summation)} \\ k_0 &\equiv \mu \sinh^2\gamma_0 = \frac{\mu(\alpha_0)^2}{1-(\alpha_0)^2}\,. \end{aligned} \qquad (4.12)$$

The ansatz solves the field equations only for equal parameters γ^A $(A = 1\ldots 3)$.[7]

Solving the stabilization equations, one obtains the moduli on the horizon in terms of the charges and the moduli at infinity. The Bekenstein-Hawking entropy takes the form

$$S = \frac{1}{4}A = 2\pi\sqrt{\left(\frac{q}{\alpha}\right)_0 D_{ABC}\left(\frac{p}{\alpha}\right)^A\left(\frac{p}{\alpha}\right)^B\left(\frac{p}{\alpha}\right)^C}\,, \qquad (4.13)$$

where

$$\begin{aligned} \left(\frac{p}{\alpha}\right)^A &\equiv \frac{p^A}{\alpha^A} = h^A\mu\cosh^2\gamma^A \qquad \text{(no summation)} \\ \left(\frac{q}{\alpha}\right)_0 &\equiv \frac{q_0}{\alpha_0} = h_0\mu\cosh^2\gamma_0\,. \end{aligned} \qquad (4.14)$$

This has the same form as the extremal entropy, with the charges (q_0, p^A) replaced by the $(\left(\frac{q}{\alpha}\right)_0, \left(\frac{p}{\alpha}\right)^A)$. Note that, unlike the extremal case, the entropy depends on the values of the moduli at infinity. In addition, the non-extremal entropy has a different functional dependence on the original charges since the parameters (α^A, α_0) depend on the charges.

The near-extremal black holes are described by adding to the extremal black holes the leading terms in μ, while holding the physical charges fixed. One gets

$$\begin{aligned} \left(\frac{p}{\alpha}\right)^A &= p^A + \frac{1}{2}h^A\mu + O(\mu^2) \\ \left(\frac{q}{\alpha}\right)_0 &= q_0 + \frac{1}{2}h_0\mu + O(\mu^2)\,. \end{aligned} \qquad (4.15)$$

We see that the near-extremal Bekenstein-Hawking entropy formula has the same structure as the extremal entropy with a modification of the charges depending on the non-extremality parameter μ and the asymptotic values of the moduli h^A. In the next section we will construct a class of horizon solutions, where this structure holds with R^2-terms, as in (1.1) and (1.2).

[7] One can relax the condition on the γ^A's by restricting the prepotential to specific choices D_{ABC}. For instance, if only D_{123} is nonzero, all γ^A's may be chosen independently. Alternatively, [7] suggests a method for finding near-extremal solutions with no restrictions but only in the near-extremal regime.

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4.2 Near-extremal $N = 2$ black holes with R^2-terms

We would like to get an explicit expression for the entropy for the near-extremal black holes with R^2-terms (3.5), as a function of the charges and the moduli at infinity. Consider black holes with one electric charge q_0 and $p^A, A = 1, 2, 3$ magnetic charges.

The bosonic part of the vector field strength F_{01}^{-I} is given by the equation of motion:

$$2(\mathrm{Im}F_{IJ})F_{01}^{-J} = G_{23I} - \bar{F}_{IJ}F_{23}^{J} + \frac{1}{2}T_{01}^{-}\mathrm{Im}\Big(F_I + F_{IJ}\bar{X}^J - 32F_{\widehat{A}I}(2C_{0101} - D)\Big) , \quad (4.16)$$

where we have introduced the dual field strength

$$G_I^{ab-} = 2i\frac{\partial(e^{-1}\mathcal{L})}{\partial F_{ab}^{I-}} \ . \quad (4.17)$$

The magnetic parts of the field strengths are obtained from Bianchi identities, and for a static spherically symmetric metric can be taken as

$$\begin{aligned} F_{23}^{I} &= \frac{1}{r^2}e^{-2U(r)}p^I \\ G_{23I} &= \frac{1}{r^2}e^{-2U(r)}q_I \ , \end{aligned} \quad (4.18)$$

where we used $g_{\theta\theta} = g_{\phi\phi}/sin^2\theta = r^2e^{2U(r)}$.

We get

$$\begin{aligned} F_{01}^{-0} &= \frac{1}{2\mathrm{Im}F_{00}}\Big(G_{230} - \bar{F}_{0A}F_{23}^{A} + +\frac{1}{2}T_{01}^{-}\mathrm{Im}\Big(F_0 + F_{0I}\bar{X}^I - 32F_{\widehat{A}0}(2C_{0101} - D)\Big)\Big) \\ F_{01}^{-1} &= \frac{1}{4\mathrm{Im}F_{21}}\Big(-\bar{F}_{21}F_{23}^{1} - \bar{F}_{23}F_{23}^{3} + \frac{1}{2}T_{01}^{-}\mathrm{Im}(F_{2I}\bar{X}^I)\Big) + \\ &\quad + \frac{1}{4\mathrm{Im}F_{31}}\Big(-\bar{F}_{31}F_{23}^{1} - \bar{F}_{32}F_{23}^{2} + \frac{1}{2}T_{01}^{-}\mathrm{Im}(F_{3I}\bar{X}^I)\Big) + \\ &\quad - \frac{\mathrm{Im}F_{32}}{4\mathrm{Im}F_{21}\mathrm{Im}F_{31}}\Big(-\bar{F}_{12}F_{23}^{2} - \bar{F}_{13}F_{23}^{3} + \frac{1}{2}T_{01}^{-}\mathrm{Im}(F_{1I}\bar{X}^I)\Big) \\ F_{01}^{-2} &= F_{01}^{-1}(1 \to 2, 2 \to 3, 3 \to 1) \qquad (\text{except "23" in } F_{23}^{I}) \\ F_{01}^{-3} &= F_{01}^{-1}(1 \to 3, 2 \to 1, 3 \to 2) \qquad (\text{except "23" in } F_{23}^{I}) \ , \end{aligned} \quad (4.19)$$

where F_{01}^{-2}, F_{01}^{-3} are obtained by cycling the indices of F_{01}^{-1}, and we have assumed the prepotential satisfies

$$\mathrm{Im}F_A = \mathrm{Im}F_{\widehat{A}A} = 0 \ . \quad (4.20)$$

With the ansatz given later for the prepotential and moduli, the terms with T_{01}^{-} in $F_{01}^{-1}, F_{01}^{-2}, F_{01}^{-3}$ are zero, and the latter further simplify to

$$F_{01}^{-A} = \frac{i}{2}F_{23}^{A} \ . \quad (4.21)$$

In addition, due to spherical symmetry:

$$F_{23}^{-I} = -iF_{01}^{-I} \ . \quad (4.22)$$

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The auxiliary field D may be determined by the constraint on the nonlinear multiplet (2.7). We will however retain D, appearing in the hatted fields (2.4) of the Lagrangian (2.3), as an independent degree of freedom. The nonlinear multiplet fields then appear only in the last line of (2.3),[8] and one can easily solve for them. The bosonic parts of the equations of motion of M_{ij} and V_a give

$$\begin{aligned} M_{ij} &= 0 \\ V_a &= 0 \,. \end{aligned} \tag{4.23}$$

In the second equation we used $e\mathcal{D}^a V_a = \partial^a(eV_a)$, which holds in the K-gauge and can be dropped in the D-gauge (2.9) as a total derivative term in the Lagrangian. The V-gauge (2.9) sets $\Phi^i{}_\alpha = \delta^i_\alpha$, and this gives $D^a\Phi^i{}_\alpha = 0$ for a solution where the SU(2) connection is zero, which we will consider. Therefore we remain with

$$D = -\frac{1}{3}R \,. \tag{4.24}$$

The area of the horizon A, the Weyl tensor C_{0101}, and the Ricci scalar R are all calculated from the metric. It remains to find solutions for the metric, the moduli $X^I(z)$, and the auxiliary field T^-_{01}. In addition, for solving the equations of motion, we will need solutions for the U(1) connection A_a and the SU(2) connection $\mathcal{V}_a{}^i{}_j$, which in the supersymmetric case could be taken as zero. We will make an ansatz for the solution on the horizon, which is an extension of both the extremal case with R^2-terms (see [1]) and the non-extremal case without R^2-terms. One may consider the ansatz of the non-extremal case for the metric (4.1), (4.4), (4.2), the modified stabilization equations (4.9) which give the moduli, and the auxiliary field (4.11), with the R^2 prepotential (2.1). However this proves to be insufficient, and since we will consider a near-extremal solution, we introduce linear μ-corrections to the fields.

Our ansatz is

$$\begin{aligned} F &= \frac{D_{ABC}X^AX^BX^C}{X^0} + \frac{D_AX^A}{X^0}\widehat{A} \qquad\qquad (4.25)\\ ds^2 &= -e^{-2U(r)}f(r)dt^2 + e^{2U(r)}(f(r)^{-1}dr^2 + r^2d\Omega^2) \\ e^{2U(r)} &= e^{-K}(1+\mu\beta_U) \\ f(r) &= \left(1-\frac{\mu}{r}\right)(1+\mu\beta_f) \\ X^A(z) &= -\frac{i}{2}x^A(1+\mu\beta_A) \\ X^0(z) &= \frac{1}{2}\sqrt{\frac{D_{ABC}x^Ax^Bx^C - 4D_Ax^A\widehat{A}(z)}{x_0}}(1+\mu\beta_0) \\ T^-_{01} &= iT^-_{23} = \left(\frac{k_0}{\alpha_0(r+k_0)} + \frac{k^1}{\alpha^1(r+k^1)} + \frac{k^2}{\alpha^2(r+k^2)} + \frac{k^3}{\alpha^3(r+k^3)}\right)\frac{1}{r}e^{\frac{1}{2}K}(1+\mu\beta_T)\,, \end{aligned}$$

[8] One may think of this last line as part of a Lagrangian multiplier which cancels the linear dependence of the original Lagrangian on D.

where

$$x^A \equiv \frac{\alpha^A p^A}{k^A} + \frac{\alpha^A p^A}{r} \qquad \text{(no summation)}$$
$$x_0 \equiv \frac{\alpha_0 p_0}{k_0} + \frac{\alpha_0 q_0}{r}\,, \tag{4.26}$$

and

$$\begin{aligned}\widehat{A}(z) &= e^{-K}\widehat{A} &(4.27)\\ &= -4e^{-K}(T_{01}^{-})^2\\ &= -4\left(\frac{k_0}{\alpha_0(r+k_0)} + \frac{k^1}{\alpha^1(r+k^1)} + \frac{k^2}{\alpha^2(r+k^2)} + \frac{k^3}{\alpha^3(r+k^3)}\right)^2 \frac{1}{r^2}(1+\mu\beta_T)^2\,.\end{aligned}$$

$\beta_U, \beta_f, \beta_1, \beta_2, \beta_3, \beta_0, \beta_T$ are finite constants,[9] and e^{-K} also contains β's. Note that besides the explicit μ-corrections above, some of the fields will also have implicit μ dependence via e^{-K} and $A(z)$.

In addition we assume

$$A_a = 0$$
$$\mathcal{V}_{a\ j}^{\ i} = 0\,. \tag{4.28}$$

The vanishing of the SU(2) connection implies also $Y_{ij}^I = 0$ [10].

For our ansatz to constitute a solution, it must satisfy the equations of motion on the horizon for the metric, the moduli $X^I(z)$, the auxiliary field T_{01}^-, and the U(1) connection. The equation of motion for the SU(2) connection is always satisfied by the vanishing SU(2) connection, for a bosonic background and with our choice of V-gauge (also assuming no hyper-multiplet scalars). This is since the SU(2) connection and its derivatives then appear in the Lagrangian always at least in quadratic form. In general the above ansatz is not a solution to the equations of motion. However, we have found that it may constitute a near-extremal horizon solution if we require equal boost parameters and $D_{ABC}p^Ap^Bp^C = 0$. The latter condition on the charges implies the vanishing of the classical horizon area in the extremal limit.

In the near-extremal regime we linearize the algebraic equations of motion (after substituting the ansatz) in the small expansion parameter $\mu \ll 2k^A, 2k_0$. This must be done after taking the horizon limit $r \to \mu$, since for a small but finite μ we want to have two topologically distinct horizons. Also, we must remember that the boost parameters α^A, α_0 depend on μ with constant k^A, k_0:

$$\alpha^A = \sqrt{\frac{k^A}{k^A+\mu}} \qquad \text{(no summation)}$$
$$\alpha_0 = \sqrt{\frac{k_0}{k_0+\mu}}\,. \tag{4.29}$$

[9] With more general r-dependent corrections, one has to note that the location of the horizon may change.

In our linear approximation the shifted charges are

$$\left(\frac{p}{\alpha}\right)^A = p^A\left(1+\frac{\mu}{2k^A}\right) + O(\mu^2) = p^A + \frac{1}{2}h^A\mu + O(\mu^2) \qquad \text{(no summation)}$$
$$\left(\frac{q}{\alpha}\right)_0 = q_0\left(1+\frac{\mu}{2k_0}\right) + O(\mu^2) = q_0 + \frac{1}{2}h_0\mu + O(\mu^2)\ . \tag{4.30}$$

Here h^A, h_0 do not necessarily correspond to the asymptotic values of the moduli, since our ansatz is only shown to constitute a solution on the horizon. Next, we choose equal boost parameters: $\alpha_0 = \alpha^1 = \alpha^2 = \alpha^3$. Without R^2-terms this would be the non-extremal version of the double-extremal black hole. Denote: $k \equiv k_0 = k^1 = k^2 = k^3$. Finally, $D_{ABC}p^Ap^Bp^C = 0$ would imply a vanishing of the classical horizon area for the extremal R-level case [11].

Under these three restrictions our ansatz (4.25) solves the field equations, with the β's for some simplified cases given in appendix A. We note that we have also found solutions with $D_{ABC}p^Ap^Bp^C \neq 0$, where the entropy does not have a simple form. In appendix B we comment on the derivation of the metric field equations.

For the above ansatz, the area of the horizon, the Ricci scalar and the Weyl tensor on the horizon read

$$\begin{aligned} A &= 4\pi\mu^2 e^{-K(r=\mu)} \\ R &= \frac{\mu\beta_f}{8\sqrt{q_0 D_A p^A}} + O(\mu^2) \\ C_{0101} &= -\frac{\mu\beta_f}{48\sqrt{q_0 D_A p^A}} + O(\mu^2)\ . \end{aligned} \tag{4.31}$$

Substituting the solution in the generalized entropy formula for the near-extremal R^2 case (3.5) yields the explicit entropy:

$$S = 32\pi\sqrt{q_0 D_A p^A}\left(1+\frac{\mu}{2k}\right) + O(\mu^2) \tag{4.32}$$
$$= 32\pi\sqrt{\left(\frac{q}{\alpha}\right)_0 D_A \left(\frac{p}{\alpha}\right)^A}$$
$$= 32\pi\sqrt{\left(q_0+\frac{1}{2}h_0\mu\right) D_A \left(p^A+\frac{1}{2}h^A\mu\right)} + O(\mu^2)\,, \tag{4.33}$$

where we must choose the signs of the charges such that the result is real. Note that due to the assumption of equal boost parameters, $h^A = p^A\frac{h_0}{q_0}$. This has the same form of the corresponding extremal R^2 case where $D_{ABC}p^Ap^Bp^C = 0$, with the charges q_0, p^A substituted by the shifted charges $\left(\frac{q}{\alpha}\right)_0 \left(\frac{p}{\alpha}\right)^A$. This is similar in fashion to the transition from the R-level extremal entropy to the near-extremal entropy. It would be interesting to compare the obtained expression for the entropy, to a corresponding microscopic statistical entropy, which is currently unknown.

As in the extremal R^2 case, we have retained only the tree-level α' F-terms, requiring that the large volume approximation is valid near the horizon by imposing: $|q_0| \gg |p^3| \gg 1$.

If the prepotential does not contain higher powers of $\widehat{A}$, the condition of large charges may be used to constrain any R^4 or higher D-term corrections. However, we cannot rule out other contributions to the field solutions and entropy coming from D-terms in the R^2-level.

The Hawking temperature for our static spherically symmetric black hole is given by

$$T = -\frac{\partial_r g_{tt}}{4\pi\sqrt{-g_{tt}g_{rr}}}\bigg|_{horizon} = \frac{\mu}{64\pi\sqrt{q_0 D_A p^A}} + O(\mu^2) \ . \tag{4.34}$$

In this approximation, replacing the charges with the shifted charges does not change the result.

Since the solutions have been constructed only on the horizon, and without the supersymmetry property, one still needs to analyze whether an interpolating solution exists which smoothly connects the horizon to asymptotically flat space. This is a prerequisite for the existence of a corresponding black hole and for the validity of the Wald entropy formula (3.1).

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A. Solutions of the field equations

Following are the explicit solutions for the β's of (4.25) which satisfy the equations of motion, under the discussed restrictions. All β's must satisfy: $|\beta| \ll \mu^{-1}$. In all the following, β_U is a real free parameter which rescales the Kähler potential relatively to the metric (which is actually a gauge freedom). Our calculations were done using Maple with GRTensor.

(i) For the case $D_{113} = D_{133} = D_{223} = D_{233} = D_{123} = D_1 = D_2 = 0$:

$$\begin{aligned}
\beta_f &= -\frac{3D_{333}p^3p^3}{256kD_3} \\
2\beta_0 &= \beta_f \\
2\beta_1 = 2\beta_2 &= \beta_f - \beta_U \\
2\beta_3 &= \beta_f - \frac{1}{k} - \beta_U \\
2\beta_T &= \frac{1}{k} - \beta_U \ .
\end{aligned} \tag{A.1}$$

(ii) For the case $D_{112} = D_{122} = D_{223} = D_{233} = D_{222} = D_{123} = D_2 = 0$:

$$\begin{aligned}
\beta_f &= \frac{3X^2}{256kY} \\
2\beta_0 &= \beta_f \\
2\beta_1 &= \beta_f - \frac{D_3p^3X}{kY} - \frac{1}{k} - \beta_U
\end{aligned}$$

$$
\begin{aligned}
2\beta_2 &= \text{anything} \\
2\beta_3 &= \beta_f + \frac{D_1 p^1 X}{kY} - \frac{1}{k} - \beta_U \\
2\beta_T &= \frac{1}{k} - \beta_U \,,
\end{aligned} \tag{A.2}
$$

where

$$
\begin{aligned}
X &= \frac{1}{2}\left(D_{333}p^3p^3p^3 + D_{133}p^1p^3p^3 - D_{113}p^1p^1p^3 - D_{111}p^1p^1p^1\right) \\
Y &= \left(D_1 D_{333}p^3p^3 + D_1 D_{133}p^1p^3 + D_3 D_{113}p^1p^3 + D_3 D_{111}p^1p^1\right)p^1p^3 \;.
\end{aligned} \tag{A.3}
$$

Note that here it is assumed that $Y \neq 0$ and

$$
D_{333}p^3p^3p^3 + 3D_{133}p^1p^3p^3 + 3D_{113}p^1p^1p^3 + D_{111}p^1p^1p^1 = 0 \;. \tag{A.4}
$$

(iii) For the case $D_{112} = D_{122} = D_{113} = D_{133} = D_{223} = D_{233} = D_{111} = D_{222} = D_{333} = 1$, $D_{123} = -\frac{7}{2}$, and $p^1 = p^2 = p^3$:

$$
\begin{aligned}
\beta_f &= 0 \\
2\beta_0 &= 0 \\
2\beta_1 = 2\beta_2 = 2\beta_3 &= -\frac{1}{k} - \beta_U \\
2\beta_T &= \frac{1}{k} - \beta_U \;.
\end{aligned} \tag{A.5}
$$

B. Derivation of the metric field equations

In order to simplify the derivation of the equations of motion, we write the Lagrangian in a form which is explicit in the scalar degrees of freedom. There are some subtleties regarding the degrees of freedom of the metric. Here we will identify these degrees of freedom and how they should be accounted for in the computation.

Let $\mathcal{L}(\psi, \partial_\mu\psi, \partial_\mu\partial_\nu\psi)$ be a Lagrangian density depending on the scalar field ψ and its first and second space-time derivatives. The equation of motion for ψ is given by the Euler-Lagrange equation:

$$
\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right) + \partial_\mu\partial_\nu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\partial_\nu\psi)}\right) = 0 \;. \tag{B.1}
$$

In our case, the action contains curvature tensors which are built from second order derivatives. Thus we need to take the full second order variation. Alternatively, one may integrate the action by parts, and take the usual first order variation.

We assume a static and spherically symmetric metric. A general form of such a metric is

$$
ds^2 = -e^{-2U_1(r)}dt^2 + e^{2U_2(r)}dr^2 + e^{2U_3(r)}r^2 d\Omega^2 \;. \tag{B.2}
$$

Correspondingly, we will get three equations of motion for $U_1(r), U_2(r), U_3(r)$. Any metric has only two real degrees of freedom: $16 - 6$ (symmetric components) - 4 (Bianchi identities) - 4 (coordinate redefinitions) = 2. So our static and spherically symmetric metric really contains only two independent r-function degrees of freedom. Thus one of the three equations of motion will be redundant.

We will explain why the above policy is nevertheless advantageous. One may set e.g. $U_2(r) = U_1(r)$ by a redefinition of the r-coordinate. This choice of "gauge" may result in a trivial equation of motion for $U_1(r)$, leaving us with only one independent equation of motion. The missing equation of motion has to be obtained from the requirement that the action is invariant under the choice of gauge. I.e. the variation of the non-gauged-fixed action with respect to $U_2(r)$ must vanish. This is also known as a Hamiltonian constraint. However, this is just the original equation of motion for $U_2(r)$ that we threw away by the gauge fixing. Thus we will simply retain all three degrees of freedom in the metric, which will give two independent equations of motion.

One may now be concerned about other gauge fixings implicit in the choice of coordinates of (B.2), e.g. vanishing off-diagonal components or $g_{\theta\theta} = g_{\phi\phi}/sin^2\theta$. However, our metric is the "maximally general" metric preserving the assumed isometries of the solution, namely staticity and spherical symmetry [12]. For such a solution, the equations of motion corresponding to the trivial metric components would be automatically satisfied and would not yield new constraints.

In order to be consistent with the notation of our solution (4.25), we will actually use the metric:

$$ds^2 = -e^{-2U_1(r)}f(r)dt^2 + e^{2U_2(r)}f(r)^{-1}dr^2 + e^{2U_3(r)}r^2d\Omega^2 \,, \tag{B.3}$$

where $f(r)$ is given. The solution to the equations of motion is given by

$$U_1(r) = U_2(r) = U_3(r) = U(r) \,, \tag{B.4}$$

where $U(r)$ is given. When deriving the equations of motion, we must retain the separate degrees of freedom of the metric.

The fields F_{ab}^{-I}, T_{ab}^{-} in our solution, are given as the anti-selfdual parts written with tangent space indices. In this form, these fields contain metric components, while the metric-independent fields are $F_{\mu\nu}^I, T_{\mu\nu}$. Let us denote by $F_{01}^{-I}(r), T_{01}^{-I}(r)$ the $(0,1)$ components of these fields as given in our solution (4.19), (4.25), before we explicitly introduced the separate metric degrees of freedom. When these fields appear in the Lagrangian explicitly (including via the hatted fields (2.4)), they should be rewritten as

$$\begin{aligned} F_{01}^{-A} &= iF_{23}^{-A} = e^{2U(r)-2U_3(r)}F_{01}^{-A}(r) \\ F_{01}^{-0} &= iF_{23}^{-0} = e^{U_1(r)-U_2(r)}F_{01}^{-0}(r) \\ T_{01}^{-} &= iT_{23}^{-} = e^{U_1(r)-U_2(r)}T_{01}^{-}(r) \ . \end{aligned} \tag{B.5}$$

Alternatively, one may work with the $F_{\mu\nu}^I, T_{\mu\nu}$ form and put appropriate projection operators in the Lagrangian.

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